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13. ABSTRACT (Maximum 200 words)

Theoretical work focused on solar and stellar convection by calculating models of compressible, magnetized convection in two and three-dimensions. The treatment of radiation transfer in the convection models was improved. Magnetic convection on all scales was studied to determine both the role of the fine scale magnetic structure and the sun's global magnetic fields and their relation to the solar cycle. Progress has been made in understanding turbulent magnetic diffusion which exerts a strong influence on the formation of magnetic field structures on all scales. The experimental work involved a balloon flight of the Solar Disk Sextant in New Mixico and the subsequent data reduction and analysis. Results showed the capability of the instrument to detect variations of the solar diameter at the few milli arcsecond level. This result fulfilled the design goal of the experiment.

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Annual Technical Report for grant N. AFOSR-91-0053, for the period 10/15/90 to 10/14/91

Principal Investigator: Sabatino Sofia
Research Title: Development of a System for Accurate Forecasting of Solar Activity. (subject areas: solar convection, influence of radiation, influence of rotation and magnetic fields, etc.)

Scientific Summary

This document reviews the accomplishments of the work supported by the AFOSR and carried out at the Yale University Center for Solar and Space Research, with collaboration by scientists at the Laboratory for Atmospheres/Solar Radiation Office at NASA/GSFC, over the last year.

Our work in this period consisted of primarily a theoretical component, with an associated experimental component whose objective is the validation of the models developed. The theoretical work focuses on solar and stellar convection by calculating models of compressible, magnetized convection in two and three–dimensions. Since the solar surface has extremely complex magnetic activity (sunspots, active regions etc.) the treatment of magnetic fields in a self consistent manner is vital to a better understanding of both the surface and internal dynamics. Since the effects of radiative transfer are also important at the solar photospheric level, we are also improving our treatment of radiation in the convection models.

This project will further improve our understanding of solar internal dynamical processes, the current solar rotation rate, and the solar dynamo. We are studying magnetic convection on small/intermediate and large scales in an attempts to determine both the role of the fine scale magnetic structure and the Sun's global magnetic fields and their relation to the solar cycle(s). The overall outcome may then be incorporated into the long term global modeling of the Sun and solar type stars. Besides increasing our understanding and ability to forecast the size and timing of the activity cycle, this will also address the theory behind solar variability on decades to centuries timescales of importance to global change and climate.

Our calculations, are computationally intensive and require, besides good workstations, supercomputer resources which we have obtained both at NASA and NSF supercomputer centers.The technique is based on the ADISM (Alternating Direction Implicit on Staggered Mesh finite difference) method which has proven to be computationally efficient and vectorizable on a variety of supercomputers.

Along the way, we have made substantial progress in understanding turbulent magnetic diffusion whose form and magnitude is of great uncertainty as it exerts a strong influence on the formation of magnetic field structures on all scales. This development could also be of use in other applications of space physics.

In addition, as observational techniques improve, more detailed information on the surface structure is uncovered. Present theoretical models are now able to model the general turbulent motions in the Sun's outer layers, but better quantitative comparison between theory and observation is required. especially when studying features related to the solar magnetic field. Our work emphasizes these comparisons.

This year past, our work has continued in a number of related areas:

- Further development of models of solar compressible convection, studying motions on the small (granular), intermediate (mesogranular) and large (supergranular) scales (P. A. Fox with K. L. Chan of GSFC and S. Sofia of Yale).
- Investigation of the details of solar synthetic spectral lines using improved model atmosphere calculations with input from simulations of the solar surface (Yong-Cheol Kim: Yale graduate student, P. A. Fox, P. Demarque, K. L. Chan and S. Sofia).
- Investigation of the so-called overshoot region at the base of the solar convection zone. This is a very important component in the general understanding of solar internal structure (P. A. Fox, Yong-Cheol Kim, P. Demarque and S. Sofia).
- Development of three-dimensional models of the interaction of convection and magnetic fields in the solar interior and refinement of our understanding of the processes involved (P. A. Fox, M. L. Theobald: Yale and S. Sofia).
- Further development of an understanding of the general properties of convective transport in stars (T. J. Lydon; graduate student at Yale, P. A. Fox, K. L. Chan and S. Sofia).

The experimental work in the last year involved a balloon flight of the Solar Disk Sextant, which took place in Fort Sumner, NM in October 1990, and the subsequent data reduction and analysis. These results showed the capability of the SDS to detect variations of the solar diameter at the few milli arc s level, thus fulfilling the design goal of the experiment.

Methodology/ Major Computational Techniques

Our strategy is to use the Alternating Direction Implicit on Staggered Mesh method (ADISM) to compute the thermal relaxation of the fluid, and switch to the explicit Adams-Bashforth method afterwards. Total thermal relaxation of a convection zone is vital for studying the structure of a convection zone (Chan and Serizawa 1991). The thermal relaxation of a convective layer, even with moderate depth, takes a very long physical time. To obtain consistency between the convective process and the structure of the convection zone, it is necessary to integrate the model beyond this time scale. Our strategy is to integrate the model through this phase as fast as possible, even if the method is not very accurate in describing short time scale transients. We switch to a temporally more accurate scheme after the relaxation.

The ADISM method obtains its efficiency through avoiding the severe time step restriction imposed by the CFL condition for a compressible fluid. This method uses a formal linearization (Taylor series expansion in time) for the non-linear terms and splits a multidimensional problem into the non-iterative solution of several block tri-diagonal matrices which can be solved efficiently (see Warming and Beam 1977, and references therein). Compared to other implicit methods, a very important advantage of the ADI approach is that its

CPU time per step grows linearly with the number of grid points, just as for explicit methods. The ext. nsion of the ADISM technique to solve the equations of fully compressible magnetohydrodynamics is described in a paper by Fox, Theobald-and Sofia (1991).

The staggered mesh was introduced to prevent the development of unstable two-grid-interval waves in the direction of gravity (Chan and Wolff 1980). It also avoids redundant computational efforts (Messinger and Arakawa 1976) and makes the handling of boundary conditions easier. For computing convective flows, the ADISM scheme has been used with time steps 1 - 2 orders of magnitude larger than the stability limit of explicit methods. Even though each implicit time step requires two to three times as many arithmetic operations as an explicit step, the gain in computational efficiency is still quite subtantial (typical a factor of 3–5 in applications). Furthermore, such gains can be preserved in vector machines. The ADI scheme has the nice property that in each spatial direction, the split differential operator (and the associated matrix) only couples array indices corresponding to that particular direction; indices for other directions are untouched. Therefore, an operation along one direction can be treated as a vector operation with vector length equal to the total number of grid points in the other directions. In the CRAY Y-MP, our 3D code can run at a sustained rate of 110 MFLOPS (single processor). We have also taken advantage of the microtasking and multitasking capabilities of the CRAY which can be particularly suited to our ADI technique and helps to reduce memory requirements of the large 3-D models.

Highlights of specific projects

The most recent results from some of the projects undertaken under the sponsorship of this grant are highlighted in this section.

Compressible Convection Studies

Turbulent compressible convection is one of the most important but least understood processes in solar physics. The solar convection zone plays a dominant role in all short term (decadal to centenary scale) variations of the solar irradiance because the upper part of it has relatively short thermal relaxation time, and it is the site of (or adjacent to) the solar dynamo.

In the last few decades, a crucial portion of our perception of the Sun was built upon a heuristic theory of convection, the mixing length theory. This theory has enjoyed many successes in its application, but remained un-verified. It was not clear how reliable this theory was nor what the limitations were. The highly non-linear and non-uniform nature of solar convection has precluded analytical approaches from making concrete progress.

Only recently, the advances in computing power and techniques have opened a new door for improving our ability to understand and calculate solar convection (for example: Cloutman 1979; Nordlund 1982, 1985; Chan *et al.* 1982; Hurlburt *et al.* 1984, 1986; Chan and Sofia 1986, 1987, 1989; Stein and Nordlund 1989; Cataneo *et al.* 1990; Chan *et al.* 1991).

Our need was to develop a more accurate, yet practical way to handle convection in solar structure and evolution calculations. As a first step, we examined the basic assumptions of the mixing length theory of convection (MLT; Biermann 1932, Vitense 1953) by solving the Navier-Stokes equations in three dimensions for a compressibly, stratified plasma. This was the theme of our work in the past few years and it has been successfully performed. Some basic ideas of MLT fare quite well in the numerical tests. However, some new phenomena have been discovered (see the next section) and they introduce intriguing complications to the problem. This does not necessarily mean that the problem is becoming more difficult, but that we have to deal with a more realistic picture of the solar convection zone.

Summary of Previous Results

Three dimensional (3D) tests on the validity of the numerical approach

Even though numerical computation is a very powerful tool, it is prone to pitfalls. In general, there is no single approach or code that is suitable for computing all hydrodynamical problems in all parametric regimes. To study turbulent compressible convection in a deeply stratified convection zone, we have used an implicit numerical method whose efficiency makes it affordable to resolve the small scale heights in the top region of the convection zone (Chan and Wolff 1982). This is an important requirement if the mixing-length effects are the matter of interest. However, the price to pay for the numerical efficiency is the decrease of time-accuracy (Chan 1983). Would that create spurious numerical results? A further question is the modelling of the sub-grid-scale (SGS) turbulence viscosity for the un-resolved properties of the turbulence. We have used the Smagorinsky formula (1963) to estimate this viscosity. This formula relates the viscosity to the local strain rate of the flow, and effectively makes the viscous term non-linear in velocity. Can this added non-linearity cause artificial mixing-length effects?

A study that tested the validity of our numerical approach has been made in three spatial dimensions, and details of the calculations and results have been documented in a paper (Chan and Sofia 1986). The principal findings are as follows:

1. Unlike all previous simulations (which found that convection in even very deep stratification consists of single vertical cells that traverse the total depth of the convection zone), the vertical motions of the fluid were found to break up in the vertical direction. More precisely, the hot, upward moving parcels of fluid turn and become discoordinated in about 1–2 pressure scale heights, and the cold, downward moving fluid form funnel-shaped, twisting, columns that persist for greater depths. The vertical correlation lengths of the vertical velocity are scaled by the local pressure height. We computed identical cases with the implicit scheme, with a second-order explicit scheme, and with different time steps, and the results were essentially identical.

2. We performed calculations that used different recipes for the effective viscosity, including one with a constant kinematic viscosity; similar phenomena appear in all of them.

3. We computed cases with different spatial resolution and domain aspect ratios. The quantative results had some small dependences on these factors. With 6 to 7 grids per pressure scale height, the spatial resolution was marginally adequate.

3D tests on the validity of MLT in efficient convection

Equipped with the further understanding of our numerical approach, we proceeded to test the validity of MLT in the case of efficient convection. By efficient convection, we mean that the convective flux is much larger than the radiative flux, so that the dynamical processes are not smeared by the effects of radiation. The results of the test support two important components of the MLT (Chan and Sofia 1987):

1. The vertical correlation of the motion of the fluid elements is scaled by the pressure scale height, indicating that the basic assumption of the MLT is valid. By comparing cases with different scale height ratios, it was shown that the correlation length *only* scales with the pressure scale height, *not* the density scale height; thus a long-standing question concerning the proper scaling length for the mixing length was resolved.

2. The convective velocity and the temperature fluctuation are related to the superadiabatic gradient by formulas similar to those suggested by MLT; thus, the heat (enthalpy) flux can be computed from the superadiabatic gradient, and vice versa. Furthermore, these relations hold for cases with different fluxes and gases with different ratios of specific heats.

Quantitative relationships derived for efficient convection

Due to the existence of dynamical and thermodynamical constraints, the fluctuating variables of the turbulently convecting fluid are related. A deeper understanding of the convective process can be obtained by studying these relationships. We therefore carried out a systematic search for quantitative relationships among the root-mean-square (rms) fluctuations and the correlation functions of the variables, using numerical data from the computed models. The results were presented as a list of approximate formulas which could be used in future analytical work (Chan and Sofia 1989). The semi-empirical formulas reflect the following results:

1. The ratios between the relative fluctuations of temperature and density, between the vertical and horizontal rms velocities, between the relative fluctuation of pressure and the square of the Mach number are quite uniform as functions of depth.

2. Many of the correlation coefficients between the fluctuating variables are uniform and insensitive to the ratio of specific heats.

3. The mean vertical velocity, assumed to be zero from time to time in analytical studies, turns out to be an important quantity that describes the mean advection of thermal variables. Almost all of the single-point, second-order correlation functions of the primitive variables can be expressed in terms of the mean vertical velocity. This quantity is also a link between the rate of buoyancy work and the enthalpy flux.

In the same study, we confirmed the existence of significant, downward fluxes of kinetic energy in deep convection zones (Massaguer and Zahn 1980; Hurlburt, Toomre, and Massaguer 1984). This phenomenon is contradictory to the assumption of the MLT. Related to this flux, some new results have been obtained:

1. The downward flux of kinetic energy cannot be treated as a diffusive flux of kinetic energy. To treat it as a diffusion of the mean square velocity is also questionable. However, it is scaled by the ratio: total flux / specific heat under constant pressure.

2. The production and dissipation of the kinetic energy do not parallel each other. Production is scaled by the total flux, and the local production rate is essentially a function of the local mean variables only. The dissipation is clearly non-local; a significant amount of kinetic energy is carried away from the production region, to be dissipated in lower regions.

3. The superadiabatic gradient in the upper convection zone can be roughly computed from the total flux; the presence of the flux of kinetic energy does not introduce serious errors to the estimate.

Dissipation of shears in a turbulent convection zone

Many studies of differential rotation in a stellar convection zone assume that the action of the convective turbulence on a large-scale shear can be treated as an eddy viscosity. However, the value of this eddy viscosity is highly uncertain. We studied the effective viscosity of the convective turbulence on large-scale shears by observing the decay of shears in a numerical convection model. Both vertical and horizontal shears were studied. The following are some preliminary results (Chan, Sofia, and Mayr 1987):

1. The vertical and horizontal effective viscosities are approximately equal.

2. The effective viscosity can be roughly estimated by one-third the product of the rms vertical velocity with the pressure scale height.

These results contain a factor of two uncertainty.

During a study on the effects of the convective turbulence on large scale temperature perturbations, long-lived periodic oscillations were found. Even without the perturbation, such oscillations (with smaller but finite amplitudes) coexist with the convective turbulence. The frequencies of the modes are in very good agreement with the acoustic frequencies obtained by eigenvalue analysis (Chan and Sofia 1988).

Work in Progress

In previous studies, we have only studied ideal situations where the radiative flux can be neglected so that the "purely" hydrodynamical principles can be isolated. In such cases, the values of the superadiabatic gradient are rather small compared to those found in the radiative-convective transition region near the solar photosphere, but these transition regions are places where MLT is needed most. In these regions, the radiative fluxes are important and they can significantly affect the advection processes. To assess the validity of MLT in these applications, it is necessary to carry out tests in such regions.

Furthermore, while several numerical simulations exist for the study of solar granules (a solar surface feature), few of them satisfy the requirement of thermal relaxation. Therefore they are not reliable in providing information about the thermal structure of the solar interior – a very important piece of information for linking the solar radius to the solar energy output. We are now doing this by simulating the upper solar convection zone, including the lower photosphere. This will be done with realistic physics like tabulated equations of state and opacities for the gas. A 3D radiative transfer code based on the Eddington approximation will also be added. Computing the upper solar convection zone serves three purposes: (i) to confront theory with observation, (ii) to study the effect of radiation on convection, and (iii) to study the behavior of the flux of kinetic energy in the overshoot region (upper photosphere) and in the very deep region.

Solar granulation/mesogranulation

As part of our statistical analysis of solar granulation we compute a number of correlations between the thermodynamic and kinematic quantities. The correlation that is particularly significant for granulation is the vertical velocity – temperature fluctuation relation, written as $C[v_r, T']$, which is directly comparable to observations at the surface. There is almost a 100% correlation between the direction of upward moving flows and temperature excesses (note that observers quote the correlation to be negative because of the convention that the line of sight upward velocities are taken to be negative).

The actual correlation calculations from the numerical simulations must be carefully extracted since the high correlation (typically -0.6 to -0.9) is actually between small scale (granular: < 4 Mm) velocity fields and the temperature field. Larger scale flows lower (smear) the correlation significantly. One technique, adopted early on in the observation of granulation, is known as *moving averages.* It selects certain smaller regions (similar to a window) to do the correlations. This has the effect of changing a low correlation value of -0.30 for $C[v_r, T']$ in data taken by Richardson and Schwarzschild (1950) into a high value of -0.68 on the same set of data (Stuart and Rush 1954).

The models that we have computed and analyzed, reproduce the range of observed RMS velocities (both horizontal and vertical) in good agreement with the solar values. Typical upflow velocities are ~ 1 km sec^{-1}, downflow velocities are ~ 2 km sec^{-1} (the intergranular downdrafts), and horizontal velocities range from ~ 2 km sec^{-1} at the surface to ~ 0.1 km sec^{-1} at the lower boundary.

We also calculate RMS intensities of the granular features for a few particular time instants for each case. Probably the best representation of the observed granular pattern is given by one of our models, having a representative maximum I' of 15.8% and minimum of -12%, leading to a granular contrast of $\sim 27.3\%$. Each of these values compares very closely to the respective observations.

The most distinctive solar granulation feature is its size distribution and we can summarize some findings from our analysis of the simulations.

1. The theoretical mean horizontal scale of granulation seems to be 0.9 to 3.0 Mm which falls well within the observational range. The cut off scale for granulation is not as well defined, but is in the range 2.3 to 4.0 Mm.

2. The characteristic scales change their distribution on the order of a granulation lifetime

(4-6 minutes).

3. Averaging correlations over longer times highlights larger scale (and hence longer lived) flows.

4. The filter sizes required to extract the granulation signal also changes in time, as the cell distribution changes.

5. Autocorrelation of vertical velocity in the upper layers is only a fair representation of cell sizes. The horizontal velocity autocorrelation and vertical velocity - temperature fluctuation correlation are much better.

6. The autocorrelation of the horizontal velocity on larger scales (no filtering) suggests a mesogranular structure (of rough order: 7Mm), At present the existence of a velocity-intensity relation for this larger scale is possible and suggested in some of the simulations but further analysis is necessary.

7. This mesoscale flow is coherent over much longer timescales than the small scale (granular) flow.

The present statistical analysis provides a useful comparison between the simulations and observations. The general features seem to be reproduced surprisingly well. There are several simplifications made in the present study that constrain what features may be compared to observations. They include: the ideal gas law, which precludes the evaluation of regions of partial ionization; the simplified treatment of radiative effects, and the absence of overshooting motions into radiative regions.

An important issue is the contribution of physical effects (*e.g.* partial ionization of H) to the allowed spatial scales. Some effects are very difficult to include or are time consuming and others are not. The distinction between these effects could be crucial when deciding if detailed calculations are always necessary, for example, when performing calculations of stellar convection zones. Another site of uncertainty is the lower boundary of the domain. Since we are forced (by computational limitations) to truncate our domain in the interior of the convection zone. The influence of the lower boundary in our analysis of the features near the surface appears to be small. Our future work may investigate possibilities such as imposing the incident heat flux not just uniformly but perhaps with a characteristic spatial variation (related to the flow scales) or randomly distributed.

The present analysis of the larger scale flow is hampered by the size of the domain, which was chosen for an analysis of granulation. In addition, the time averaging sequences need to be longer to determine if an individual set of characteristics exist for the large scale (*i.e.* size, lifetime, intensity contrast perhaps, *etc.*) or whether they are controlled by the granular scale flow. Further studies of vertical and horizontal flow velocities, especially with depth, are required.

Apart from the restrictions we have already mentioned, the effects of including only two spatial dimension does not diminish the very good agreement of the current models with a wide range of observations. An important task will be to define the quantitative difference that the extra dimension allows for in the present results. The study of detailed physical effects and numerical constraints is much more affordable in two dimensions but we must understand the limitations.

One area for further study is to determine the influence of the solar magnetic field on both granulation and larger scale flows. This is one area where high quality, coordinated obsrervations will be an important test of theoretical models.

The comparison of numerical simulations to observations is an important tool in tuning convection models. The analysis of other, non-observable, quantities using statistical techniques is providing a much better understanding of highly compressible, turbulent convection so that when detailed granular models are applied to, or related to, other phenomena (such as umbral granulation, stellar granulation, spectral line asymmetries, *etc.*) their limitations are understood.

Convective flows around Sunspot-like objects

We have examined a limited number of calculations of compressible convective flow around a sunspot-like object placed in the upper solar convection zone. The aim of these calculations was to discover where and how the emerging heat flux would be diverted away from the object. This problem is an important component in the understanding of variations in the Sun's radiation output over the timescales of hours to months.

In all the cases studied there was a significant increase in the kinetic energy flux and thus the lateral transport of the energy flux surrounding the object. This mainly occured below the object to allow the heat to appear at the surface, with some time delay. It also allowed it to be transported horizontally far from the object, and either be stored in the internal fluid circulations or escape from the lateral boundaries. The extent to which these three possibilities may occur depends somewhat on the size and position of the object, but for all the cases we computed the tendency was for the heat to try and re-emerge at the surface.

Most of the time the flows adjacent to the object (in all cases) were directed downwards except when a hotter than average region was diverted from just below the object to the bottom edge. In that case there was a short term upflow to bring the heat to the surface. In addition, most of the time the correlations between flow and thermodynamic quantities that usually characterize convective flow (such as the vertical velocity – temperature fluctuation relation, see Fox 1989; Chan and Sofia 1989) were not altered to any noticeable degree indicating that the readjustment to the presence of the object was on a convective rather than diffusive timescale.

Given the importance of the kinetic energy in the redistribution of the heat flux, it is not surprising that previous studies found markedly different behavior. Most of the previous work relied only on a diffusion treatment of the heat flow, or used linearized equations, and thus excluded the process that seems to be important in these calculations.

An important result is that the diverted heat flow was *not* totally blocked by the object. The main effect was in the time delay for redistribution. Many of the details of how and where the local flux is diverted depend on the particular depth at which the object is, and how its size compares with the local overturning (or convective) scale, which is usually some multiple of a pressure scale height.

By integrating the average surface flux in time we were able to determine that most of the blocked heat actually re-appears at the surface (after a short period of relaxation when the object is first positioned). In addition, when the object is removed, the layer reacts very quickly and after about one hour the surface flux almost returns to its solar value. Another few hours is required though for the layer to fully relax indicating that the object caused some storage of heat within the layer and some heat flux to leave the domain laterally. Naturally, these timescales depend on the size and position of the blocking.

Some obvious improvements to the present formulation may include a better treatment of the surface boundary conditions and better radiative transfer within the domain. The size limitation of the object compared to the domain size is a major factor in modeling real solar features with any degree of realism.

Because the sunspot-like object is really magnetic in nature and the convective flow closely surrounds it, it is likely that there will be some conversion between internal, kinetic and magnetic energy (Fox, Theobald and Sofia 1991). This process has important consequences for atmospheric heating and related phenomena and it may also alter the specific intensity of the object (smooth it out perhaps). In particular in localized regions where the magnetic field and current are large, the Poynting flux can be an important component of the total flux. The current assumption of the passive existence of the object is only somewhat valid based on surface observations; however the lack of any knowledge on subsurface fields can be seen as an obvious drawback.

Work is in progress, which includes the effects of magnetic fields explicitly. Unfortunately, our understanding of the statistical properties of highly time dependent magnetoconvection is far below that of the non-magnetic case. Thus, we also intend to pursue the present approach, as we expect to learn a great deal about the heat flow distribution from refined analysis techniques and in much larger convective regions, perhaps in three dimensions and at different depths in the convection zone.

Interaction of Convection and Magnetic Fields

The emphasis of our early work on this subject has been to test our formulation and numerical solution of the relevant equations and to add to the qualitative understanding of magnetized compressible convection. This understanding is still far from complete.

Our results indicate that the time for thermal relaxation of a convective layer with magnetic fields is primarily influenced by the mean magnetic flux that permeates the layer. This result is consistent with the hydrodynamic case, in which the mean thermodynamic properties are important, and the system evolves irrespective of a random initial velocity field (recall that the magnetic pressure is part of the total pressure). For solar and stellar applications this is a very important result, since mean magnetic fluxes are measurable quantities, even though the structure of the field is not. In addition, since the relaxation time seems to decrease as the magnetic flux increases, the laborious task of reaching a statistically stationary configuration is eased. This may have important consequences in stars where the entire convection zone can be modeled in a single simulation, *e.g.* F-stars, and thus the evolution of the magnetic field could be studied in detail.

A secondary effect on the relaxation, once the field strength passes a critical value (which depends on local conditions, such as the gas pressure), is the orientation of the mean field.

In this situation, predominantly vertical fields provide more rapid relaxation than do predominantly horizontal fields. The details of the internal (initial) structure do not seem to be important for the cases we have considered but as the field strength increases, the type of boundary conditions, *i.e.* the structure of the externally imposed magnetic field is likely to be important. An even smaller effect on the relaxation is due to magnetic resistivity.

The presence of waves (acoustic, magnetoacoustic and Alfvén) is an important part of all our results. Certainly, the increasing broad band features in the energy power spectra with increasing field strength, suggests complex combinations and interactions between the waves. The presence of the abovementioned features raises a number of important questions.

For global modeling, such as solar cycle surface features and the dynamo problem, it is important to know the role of wave–like phenomena. In particular the observational separation between large scale waves (e.g. the torsional oscillation pattern, (Howard and Labonte 1981)) and small scale waves (e.g. oscillations in sunspots, Thomas 1981) could be a difficult one. We must know how important waves are, whether we can ignore them, or whether the details of even the smallest scale features must be included. This will be one emphasis for our future studies.

When modeling specific features on small scales, the details of the waves and their contribution to observable quantities can be important. The difficulty that we encounter in simulations is always one of realism, particularly when dealing with magnetic fields and imposing boundary conditions. The generation and dissipation of waves within the domain is likely to be influenced by what happens at the boundaries. Except in the restrictive case of a perfectly conducting boundary, magnetic field lines are not necessarily confined to the domain and may cross the boundary with an arbitrary orientation. The usual boundary conditions on a velocity field, unless overshooting is incorporated, prevent flow normal to the boundaries. As a consequence, magnetic fields may be swept along the boundary by tangential flows, such as those in the present results. A combination of the flow and magnetic boundary conditions thus places constraints on the boundary field orientation. An understanding of boundary effects is essential when interpreting wave-like behavior. We have yet to fully address this issue, however our results indicate that as the strength of the magnetic field increases so does the influence of the boundary on the interior. This is an area of concern for numerical simulations that must be addressed. Some work that allows for transmissive (or radiation) boundaries has already commenced, but deals only with the wave–like nature of the equations. The complicating effects of dissipation, leading to parabolic equations, is yet to be dealt with.

An analysis of the nature of compressible convection in the absence of magnetic fields (Chan and Sofia 1989) has provided some important insights into the mean properities of deep and efficient convection. When the effects of magnetic fields are introduced into the simulations, especially with increasing field strength, the degree of non-linear interaction increases. The present results indicate that both mean and fluctuating quantities are influenced and thus it seems that the general properties of convection as the primary heat transport mechanism are likely to change. Many of our results support that notion; for example, the change in the distribution of convection patterns with increasing field strength. The search for statistical relationships between mean convective and magnetic properties is a logical next step.

As the magnetic field strength is increased there is a clear tendency to form identifiable cellular structures, especially in the case of predominantly vertical fields. We would expect a similar effect to occur in three dimensions; however, the form of the "cells" may be different. This tendency toward cells is important, since existing simulations that do not include magnetic fields exhibit distinct, non-cellular flows: the upflows are broad and the downflows are narrow but streamlines are not necessarily closed. In the case of a predominantly horizontal field, the small scale flows that are generated are quite different to their vertical counterpart. We expect that in three dimensions their evolution will be even less regular and thus may enable the distinction between the general orientation of regions with stronger magnetic fields by comparison with surface layer observations. The changing nature of convective flow with mean orientation could have important consequences for understanding phenomena related to stellar convection zones, for example, helping to explain stellar dynamos and surface abundance inhomogeneties and anomolies.

Apart from the effects of increasing magnetic field strength, the magnetic resistivity influences the details of the concentration and expulsion of magnetic flux and magnetic field line reconnection. In a simulation where the structure of the field is important, an uncertainty in the value (let alone functional form) of the resistivity could lead to an inaccurate representation of the field structure. The present qualitative models do not allow the distinction between molecular resistivity (Spitzer 1962) and an artificially increased resistivity due to the effects of turbulence, the latter usually being a crude estimate of a standard diffusion coefficient (*i.e.* $\alpha V \ell$, where $\alpha \sim \mathcal{O}(1)$, V and ℓ being typical velocity and length scales). In the case of the Sun, it has long been argued that the turbulent value is more suitable than the smaller molecular value based on the observed decay of solar magnetic surface features.

Clearly an ignorance of the magnetic resistivity can not be tolerated when dealing with detailed numerical simulations. One advantage of our numerical scheme is that it does not require the addition of artificial or enhanced diffusion coefficients to maintain numerical stability. This means we are able to investigate physically meaningful dissipation processes. In the case of the fluid viscosity, a primitive understanding of turbulence lead to formulations (Smagorinsky 1963; Deardorff 1971) such as the SGS viscosity we use in our models. We are currently investigating an extension of this procedure to formulate an equivalent expression for the magnetic resistivity (Theobald, Fox and Sofia 1991).

As we (and others) have already indicated, the ADISM method is very suitable for studying simulations of compressible magnetohydrodynamics. The staggered mesh formulation removes the need for artificial viscosities and allows us to use the vector potential formulation without incurring a loss of accuracy in computing second derivatives.

Although the results of our present work are restricted to two spatial dimensions, many of the comments we have made will likely also apply to the three dimensional case. This expectation is based on the fact that in the hydrodynamic case, while some differences exist between two and three dimensions, many of the main features are common. Another future task will be to compare the current simulations with those in three dimensions.

An Improved Convective Energy Transport Formulation

After the analysis of a series of numerical simulations of turbulent, compressible convection, Chan and Sofia (1989) provided a list of approximate formulae describing quantitative relationships between thermodynamic variables (such as pressure and temperature) and flow variables (such as velocities). Many of the Chan and Sofia relationships were formulated for ease of comparison with the mixing length theory (MLT) of convective heat transport. A thorough comparison was made between the numerical relationships and mixing length theory when both are applied to the convection zone of a standard solar model. The long term goal of such research is to remove the MLT entirely from solar models.

In order to construct a solar model with one solar mass and the measured solar luminosity ($L_\odot$) at the solar age (4.5 Gyrs), the initial helium abundance (Y, for a fixed heavy element abundance Z) must be adjusted. To first order, a larger helium abundance yields a more luminous star. Throughout the comparison we present here, $Y = 0.2792$. In order to fit the one solar mass star to the observed solar radius ($R_\odot$), MLT requires similar adjustment of the free parameter α, known as the mixing length parameter (ratio of mixing length to pressure scale height). Although the exact role of the mixing length parameter will be discussed subsequently, a larger value of α yields a smaller star. In all of our reference models we adopt $\alpha = 1.24$.

This fitting process reveals the major drawback of MLT (first highlighted by Demarque and Percy 1964); the theory contains one free parameter (α) which can only be calibrated with the Sun, the sole star with an accurately measured radius. Although Y can be crudely estimated by other means, there is no *a priori* means of estimating α. MLT assumes that α is approximately unity, but in application, the value of α varies from star to star and from model to model. Such variance is a second major drawback of MLT (although many consider it an advantage). Different researchers, using different stellar structure codes, with different treatments of physical processes, are all able to produce a "solar model" by simply adjusting α. In a sense, α is a panacea; the salient, underlying features of different solar models can be glossed over by adjusting it. The extreme flexibility of MLT makes it a widely used technique but renders detailed comparison between models meaningless.

A Revised Solar Model

In order to make a useful comparison between MLT and the Chan and Sofia relationships, a revised solar model was created free of MLT. The goal of MLT is to solve for the temperature gradient within a convective region. This is accomplished by setting the solar flux of energy into a layer ($F_{\rm Total}$) equal to the sum of a convective flux ($F_{\rm conv}$) and a radiative flux ($F_{\rm rad}$):

$$F_{\rm Total} = F_{\rm conv} + F_{\rm rad}. \tag{0.1}$$

Radiation is treated using the diffusion approximation and thus, once the MLT expression (??) is substituted, the only unknown is ∇, the logarithmic gradient, expressed in terms of a cubic equation. In our revised solar model, $F_{\rm conv}$ has been replaced by the sum of an enthalpy flux ($F_{\rm ep}$) and a kinetic energy flux ($F_{\rm ke}$):

$$F_{\rm Total} = F_{\rm ep} + F_{\rm ke} + F_{\rm rad}. \tag{0.2}$$

From the numerical simulations, relationships R21 and R25/26 provide expressions for F_{ep} and F_{ke}:

$$\text{R21} \quad F_{\text{ep}} \approx 0.72(c_p\mu_t/R^*)\rho V_z''^3 \tag{0.3}$$
$$\text{R25/26} \quad F_{\text{ke}} \approx -0.64\rho V_z''^3. \tag{0.4}$$

Once these terms are substituted and R23 is included, the result is also a cubic equation for $\Delta\nabla$. Consequently this formulation directly replaces the MLT procedure in calculating the temperature gradient in convective regions.

If we now recompute a stellar evolutionary model of the Sun we can compare the result with both previous models and the observed constraints. Although the revised model does not exactly converge to one solar radius and one solar luminosity, the agreement between the two sets of tracks is quite good. The revised solar model yields a star which is slightly small (0.969 $R_\odot$) rather than much too small, as one might have expected from the earlier calculation of $\alpha_{\text{eff}} = 2.3$.

The increased sensitivity, and thus lack of adjustability, in the new solar models provides an excellent testing ground for other physical processes. For example, the addition of an empirical model atmosphere instead of a conventional (Eddington type) analytic expression, provides a solar model with the solar radius ($R = 1.003R_\odot$) and luminosity ($L = 1.001L_\odot$) in very close agreement with their measured values. In fact, with an improved treatment of convective energy transport, it should now be possible to unravel the complexities of the radiative layer near the solar surface ($T(\tau)$ relation, *etc.*).

Other Stars

Although the results are still preliminary, a necessary consistency check for the revised theory of convection is the application to nearby stars. For example, Alpha Cen A is slightly more massive than the Sun (1.09 $M_\odot$), while Alpha Cen B is slightly less massive (0.90 $M_\odot$). Both stars have a slightly higher abundance of heavy elements than the Sun, and the stars are approximately as old as the Sun (4.5±0.5 Gyrs). Detailed modelling of both stars using MLT will soon appear in a paper by Edmonds *et al.*, 1992. Applying our revised model of convection to Alpha Cen A, we obtain both radius and luminosity within the range of observational uncertainty. Similar results apply for our Alpha Cen B model. Even though such results are preliminary, nevertheless, they do provide encouraging evidence that our revised theory of convection is widely applicable.

The Solar Disk Sextant

The SDS is a space based instrument, developed over the last decade in collaboration with scientists at NASA/GSFC, whose purpose is to measure the solar size, and its variations, with unparallelled accuracy. The scientific motivation of this instrument is to measure the solar oblateness, the long period (particularly g-mode) oscillations, and the long period (decades or longer) variations of the solar diametern related to the solar dynamo (Sofia et al, 1991). To accomplish these objectives, the instrument should be able to measure diameter variations with an accuracy of a few milli arc s over minutes. This capability was demonstrated in a balloon flight of the instrument carried out from Fort Sumner, NM, in October 1990 (Maier, Twigg and Sofia, 1992). In this flight, we were able to detect the apparent change of the solar size due to the orbital eccentricity of the Earth over two separate 20 min observing runs. The amplitude of this effect over the observing period is only about 5 arc s. We also demonstrated the need to build the objective wedge by the technique of optical contacting, since sperings are not adequate to keep the wedge components stable with the required accuracy. We are presently constructing an optically contacted wedge to carry out another flight in October 1992.

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